



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$+(1n^{n+1}+2^{n+1}+3^{n+1}+4^{n+1}+\dots+n^{n+1})a_n.$$

Transposing the last quantity in parenthesis, reducing, and replacing the values of a_1 , a_2 , a_3 , we get,

$$(1+2+3+\dots+n) + \frac{n}{2!}(1^2+2^2+3^2+\dots+n^2) + \frac{n(n-1)}{3!}(1^3+2^3+3^3+\dots+n^3) \\ + \dots + \frac{n(n-1)}{3!}(1^{n-2}+2^{n-2}+3^{n-2}+\dots+n^{n-2}) + \frac{n}{2!}(1^{n-1}+2^{n-1}+3^{n-1} \\ + \dots + n^{n-1}) + (1^n+2^n+3^n+\dots+n^n) = (n+1)^n - 1.$$

When $n=2$, we get $(1+2)+(1^2+2^2)=3^2-1=8$.

When $n=3$, $(1+2+3)+\frac{3}{2}(1^2+2^2+3^2)+(1^3+2^3+3^3)=4^3-1=63$.

When $n=4$, $(1+2+3+4)+2(1^2+2^2+3^2+4^2)+2(1^3+2^3+3^3+4^3) \\ +(1^4+2^4+3^4+4^4)=5^4-1=624$.

When $n=5$, $(1+2+3+4+5)+\frac{5}{2}(1^2+2^2+3^2+4^2+5^2)+\frac{10}{3}(1^3+2^3+3^3 \\ +4^3+5^3)+\frac{5}{2}(1^4+2^4+3^4+4^4+5^4)+(1^5+2^5+3^5+4^5+5^5)=6^5-1=7775$.

When $n=6$, $(1+2+3+4+5+6)+3(1^2+2^2+3^2+4^2+5^2+6^2)+5(1^3+2^3+3^3 \\ +4^3+5^3+6^3)+5(1^4+2^4+3^4+4^4+5^4+6^4)+3(1^5+2^5+3^5+4^5+5^5+6^5)+(1^6+2^6+3^6 \\ +4^6+5^6+6^6)=7^6-1=117648$.

Also solved by ELMER SCHUYLER.

102. Proposed by J. MARCUS BOORMAN, Woodmere, N. Y.

Solve $2x+\sqrt{x^2-7}=5$.

I. Solution by COOPER D. SCHMITT, A. M., University of Tennessee, Knoxville, Tenn.; M. A. GRUBER, A. M., Washington, D. C.; J. SCHEFFER, A. M., Hagerstown, Md.; and G. B. M. ZERR, A. M., Ph. D., Chester High School, Chester, Pa.

Transposing, $\sqrt{x^2-7}=5-2x$.

Squaring, $x^2-7=25-20x+4x^2$; whence $3x^2-20x+32=0$.

Solving, $x=4$ or $\frac{8}{3}$.

Neither of these satisfies the original equation, but by writing it thus, $2x-\sqrt{x^2-7}=5$, both values will satisfy it.

II. Solution by H. C. WHITAKER, M. E., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

Denote $2x-5$ by y ; then the given equation reduces to

$$+\sqrt{(y^2+10y-3)}=-2y.$$

Divide by y , $+\sqrt{(1+10/y-3/y^2)}=-2$ (1).

But this equation is absurd, since it makes a positive square root equal to a negative number.

Assume the symbolism $\pm\sqrt{j} = -1$, j being an impossible quantity, the root of the equation $\pm\sqrt{x+1}=0$.

Square (1), $1+10/y-3/y^2=4j$, $y^2(4j-1)-10y+3=0$, from which

$$x = \frac{10j \pm \sqrt{(7-3j)}}{4j-1}.$$

III. Solution by the PROPOSER.

Transpose and square.

$\therefore 4x^2 - 20x + 25 = x^2 - 7 \dots\dots (B)$, an obvious quadratic.

Apply its roots, 4 and $\frac{1}{3}$, to the given (A); hence $2(4) + [-3] = 8 - 3 = 5$;
 $\dots\dots = 2x + \{-[\sqrt{(16-7)}]\} \dots\dots (C)$; and

$$2(\frac{1}{3}) + (-\frac{1}{3}) = 5\frac{1}{3} - \frac{1}{3} = 5 \dots\dots = 2x + \{-[\sqrt{(\frac{64}{9} - \frac{6}{9})}]\} \dots\dots (D);$$

satisfy it. Could extracting $\sqrt{x^2 - 7}$ positive here also yield roots, then (A)'s dominant quadratic (B) is bi-quadratic, which is absurd.

Also solved by P. S. BERG and CHAS. C. CROSS.

GEOMETRY.

127. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

The equation to the plane through the extremities, (x_1, y_1, z_1) , (x_2, y_2, z_2) , (x_3, y_3, z_3) , of conjugate diameters of the ellipsoid,

$$\frac{x^2}{y^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ is } \frac{x_1+x_2+x_3}{a^2}x + \frac{y_1+y_2+y_3}{b^2}y + \frac{z_1+z_2+z_3}{c^2}z = 1.$$

Solution by the PROPOSER.

If $lx+my+nz=p \dots\dots (1)$ be the required plane, we should have

$$lx_1+my_1+nz_1=p \dots\dots (2),$$

$$lx_2+my_2+nz_2=p \dots\dots (3),$$

$$lx_3+my_3+nz_3=p \dots\dots (4).$$

Solving these for l/p , m/p , n/p , we have

$$l/p = \begin{vmatrix} 1, & y_1, & z_1 \\ 1, & y_2, & z_2 \\ 1, & y_3, & z_3 \end{vmatrix} \div \begin{vmatrix} x_1, & y_1, & z_1 \\ x_2, & y_2, & z_2 \\ x_3, & y_3, & z_3 \end{vmatrix} \dots\dots (5).$$

$$m/p = \text{etc.}, \quad n/p = \text{etc.}, \dots\dots (6).$$

Reducing (5), making use of